M. Math IInd year Final examination .
Advanced Functional analysis

Answer all the 10 questions. Each question is worth 6 points.

If you are using any result proved in the class, you need to state it correctly.

1. Let X be a locally convex real topological vector space and let $A \subset X$ be a compact set. Show that the convex hull of A is a totally bounded set.
2. Let c_0 denote the space of null sequences equipped with the supremum norm. Show that the closed unit ball has no extreme points.
3. Let X be a completely metrizable topological vector space. Let $Y \subset X$ be a closed subspace. Show that the quotient space X/Y is also completely metrizable.
4. Let T denote the unit circle in the complex plane. Let $\mathcal{P}$ be space of trigonometric complex polynomials. Show that any weak*-compact convex set in $\mathcal{P}^*$ is a norm bounded set.
5. Let X be a Banach space such that the closed unit ball of X^* has only finitely many extreme points. Show that X is a finite dimensional space.
6. Let $K = \mathcal{P}([0,1])$ denote the set of regular Borel probability measures on $[0,1]$ equipped with the weak*-topology. Let $\delta : [0,1] \to K$ be the Dirac map. Show that $\Phi : A(K) \to C([0,1])$ defined by $\Phi(a) = a \circ \delta$ for $a \in A(K)$, is a surjective isometry.
7. Let X be a locally convex space and $Y \subset X$ be a closed subspace.Show that any $\Lambda \in Y^*$ has an extension to a $\Lambda' \in X^*$.
8. Let λ be the Lebesgue measure on $[0,1]$. Let $f : [0,1] \to L^2([0,1])$ be a continuous function. Show that f is a Bochner integrable function.
9. Let $(\Omega, \mathcal{A}, \mu)$ be a finite measure space. f, g be two Bochner integrable functions valued in a Banach space. Suppose $\int_E f d\mu = \int_E g d\mu$ for all $E \in \mathcal{A}$. Show that $f = g$ a.e .

10. State and prove the dominated convergence theorem for Bochner integrable functions.